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Witnessing quantum phase transition in a non-Hermitian trapped ion system via quantum Fisher information

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Abstract: Quantum Fisher information is used to witness the quantum phase transition in a non-Hermitian trapped ion system with balanced gain and loss, from the viewpoint of quantum parameter estimation. We formulate a general non-unitary dynamic of any two-level non-Hermitian system in the form of state vector. The sudden change in the dynamics of quantum Fisher information occurs at an exceptional point characterizing quantum criticality. The dynamical behaviors of quantum Fisher information are classified into two different ways which depends on whether the system is located in symmetry unbroken or broken phase regimes. In the phase regime where parity and time reversal symmetry are unbroken, the oscillatory evolution of quantum Fisher information is presented, achieving better quantum measurement precision. In the broken phase regime, quantum Fisher information undergoes the monotonically decreasing behavior. The maximum value of quantum estimation precision is obtained at the exceptional point. It is found that the two distinct kinds of behaviors can be verified by quantum entropy and coherence. Utilizing quantum Fisher information to witness phase transition in the non-Hermitian system is emphasized. The results may have potential applications to non-Hermitian quantum information technology.

Key words: PT symmetry; non-Hermitian system; quantum Fisher information; quantum criticality; ion trap

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通过量子 Fisher 信息量标度非厄米离子阱系统中的量子相变

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摘要：本文以具有增益损耗平衡的非厄米离子阱系统为研究对象，从量子参数测量角度，利用量子 Fisher 信息量标度非厄米系统的量子相变特征。通过态矢量映射方法，研究了任意两能级非厄米量子系统的一般非幺正演化规律。量子 Fisher 信息量的动力学演化在奇异点附近发生突然变化，并定量表征系统的量子临界现象。根据系统物相是否具有宇称和时间反演对称特性，可以获得两种不同行为的演化过程。在对称相区域中，量子 Fisher 信息量随时间呈现振荡特征，可获得较高的测量精度。在对称性被破坏的相区域里，它的含时变化经历单调递减过程。这两种动力学行为也被量子熵和量子相干证实。强调了利用量子 Fisher 信息来见证非厄米离子阱系统的相变。这些结论有助于非厄米量子信息技术发展。

关 键 词：PT 对称；非厄米系统；量子 Fisher 信息；量子临界；离子阱系统
中图分类号：O431.2 **文献标志码：**A **doi：**10.37188/CO.EN-2024-0017 **CSTR：**32171.14.CO.EN-2024-0017

1 Introduction

Hermitian and non-Hermitian Hamiltonian are the two types of quantum systems. It is well known that the dynamics of systems governed by Hermitian Hamiltonian are referred to as unitary. In this case, the Hermitian Hamiltonian is characterized by real energy spectrum. However, Non-Hermitian Hamiltonian system undergoes non-unitary evolution which corresponds to an open quantum system influenced by an external environment. In such systems, it provides complex eigenvalues and non-orthogonal eigenvectors. Due to the rapid development of non-Hermitian optical and micro-cavity experiments[1], a class of non-Hermitian Hamiltonian with parity-time(PT) symmetry exhibiting real energy eigenvalues have attracted numerous attentions.

It has been demonstrated that non-Hermitian systems in the parameter space usually experience the quantum criticality at an exceptional point (EP). Arising from the coalescence of both eigenvalues and eigenvectors, non-Hermitian systems with PT symmetry exhibit quantum phase transition from unbroken phase regime to broken one. In recent years, some new features at exceptional points can be observed in non-Hermitian system, such as single-mode lasing[2], enhancing optical response behaviors[3], PT-symmetry-breaking chaos[4], flexible switching for light transmission[5], loss-induced transparency[6], and EP-enhanced sensing[7-8]. Therefore, a fundamental issue is how to effectively detect and characterize critical phenomena in non-Hermitian systems[9].

In some works, various kinds of witnesses have been put forward to capture the features of quantum phase transitions, including Hilbert-Schmidt speed[10], quantum coherence[11] and entropy[12]. Although these quantities are sensitive to quantum criticality, they are theoretical evidences which are indirectly observable. To directly measure critical phenomena, we expect to apply metrological methods from the viewpoint of parameter estimation. This motivation stimulates us to explore the relationship between quantum metrology and quantum phase transition in non-Hermitian systems. It is well known that quantum Fisher information (QFI) is usually exploited to evaluate the precision of parameter estimation[13]. Consequently, an interesting question arises: how to determine the PT phase transitions from the perspective of quantum parameter estimation. In the present work, we provide the witness based on quantum Fisher information to accurately characterize signatures of quantum critical transitions in non-Hermitian ion trap systems with PT symmetry.

This paper is organized as follows. In Sec. II, a normalized density matrix in the form of state vector is used to define quantum Fisher information in a non-Hermitian PT-symmetric system. In Sec. III, we provide a general formulation to characterize the non-unitary dynamics of any two-level non-Hermitian system. By using the general approach, we investigate the dynamics of quantum Fisher information in single ion trapped system. The physical relationship between quantum parameter estimation and quantum phase transition is studied in Sec. IV. The exceptional criticality is also verified by quantum entropy and quantum coherence. Finally, a conclusion is given in Sec VI.

2　Quantum Fisher information with PT symmetry

To obtain the QFI for non-Hermitian system, let us start by reviewing some fundamental features of the QFI definition. For a quantum state ρ_θ with an unknown parameter θ, the QFI F_θ provides the ultimate precision limit according to the estimation theory. It is known as the quantum Cramér-Rao bound, $V(\theta)\geqslant 1/(nF_\theta)$ where n is the number of measurements. The large value QFI corresponds to the high precision of parameter estimation. The general expression is written as $F_\theta = Tr[L_\theta^2\rho_\theta]$ where the symmetric logarithmic derivative operator L_θ satisfies $(L_\theta\rho_\theta+\rho_\theta L_\theta)/2=\partial_\theta\rho_\theta$ and arbitrary states ρ_θ can be diagonalized[14-15]. Note that a Hermitian system governs an unitary evolution which obeys the conservation of the trace of the density matrix and the eigenvectors are orthogonal and normalized. However, the dynamics of a non-Hermitian quantum system is no more unitary and the estimated state is no longer normalized. According to the derivation[16], we need to normalize the evolved state as $\tilde{\rho}_\theta=\rho_\theta/Tr(\rho_\theta)$ where $Tr(\rho_\theta)$ is the trace of the estimated density matrix. In the condition of PT symmetry, the density matrix of a $d-$level system is Hermitian which satisfies $\rho_\theta^\dagger=\rho_\theta$, and then can be expressed as a linear superposition of a complete set of Hermitian operators. In the case of $d=2$, $\rho_\theta(t)=\dfrac{1}{2}\sum_{j=0}^{3}\lambda_j(t)\hat{E}_j$, where the coefficients $\lambda_j(t)=Tr\left[\rho_\theta(t)\hat{E}_j\right]$ are real. Here, $\{\hat{E}_j\}$=$\{\sigma_0,\sigma_1,\sigma_2,\sigma_3\}$ is a complete set of Pauli operators and $Tr(\rho_\theta)=\lambda_0(t)$ is the normalized parameter. If the normalized matrix is $\tilde{\rho}_\theta=\sum_j c_j\left|\psi_j\right\rangle\left\langle\psi_j\right|$, the QFI F_θ is written as

$$F_\theta=2\sum_{j,k}\frac{\left|\left\langle\psi_j\right|\partial_\theta\tilde{\rho}_\theta\left|\psi_k\right\rangle\right|^2}{c_j+c_k}\quad,\tag{1}$$

where the relation of eigenvalues $c_j+c_k\neq 0$ is required and $c_{1,2}=\dfrac{\lambda_0(t)\pm\sum_{j=1}^{3}\sqrt{\lambda_j^2(t)}}{2\lambda_0(t)}$, $\left|\psi_{1,2}\right\rangle=\begin{pmatrix}\lambda_1(t)-i\lambda_2(t)\\ -\lambda_3(t)\pm\sum_{j=1}^{3}\sqrt{\lambda_j^2(t)}\end{pmatrix}$ denote the eigenvalues and eigenvectors of the estimated state respectively.

3　Dynamics of a non-Hermitian system in the form of vectors

To demonstrate the dynamical behavior of QFI in a non-Hermitian system, we consider a general two-level non-Hermitian system whose Hamiltonian is written as

$$\hat{H}=\hat{H}_+ + \hat{H}_-\quad,\tag{2}$$

where a Hermitian part $\hat{H}_+=\hat{H}_+^\dagger=\sum\alpha_k\hat{E}_k$, an anti-Hermitian one $\hat{H}_-=-\hat{H}_-^\dagger=i\sum\beta_l\hat{E}_l$ and the coefficients $\{\alpha_k,\beta_l\}$ are real. The state evolution of the non-Hermitian system is governed by the time-dependent Schrodinger equation[17-18] which reads as,

$$\frac{\partial\rho(t)}{\partial t}=-\frac{i}{\hbar}\left(\hat{H}\rho(t)-\rho(t)\hat{H}^\dagger\right)=-\frac{i}{\hbar}\left[\hat{H}_+,\rho(t)\right]-\frac{i}{\hbar}\left\{\hat{H}_-,\rho(t)\right\}\quad,\tag{3}$$

here, the square bracket $[\cdot,\cdot]$ is the commutator, the curly bracket denotes the anti-commutator $\{\cdot,\cdot\}$ and the density matrix $\rho(t)$describes the state of the non-Hermitian system. By using the vector formalism $\vec{\lambda}(t)$ of the density matrix, the dynamics of the non-Hermitian system can be expressed as,

$$\frac{\partial}{\partial t}\vec{\lambda}(t)=\hat{\zeta}\cdot\vec{\lambda}(t)\quad, \tag{4}$$

where the dynamical mapping matrix is $\hat{\zeta}=\frac{2}{\hbar}\begin{pmatrix}\beta_0 & \beta_1 & \beta_2 & \beta_3\\ \beta_1 & \beta_0 & \alpha_3 & -\alpha_2\\ \beta_2 & \alpha_3 & \beta_0 & -\alpha_1\\ \beta_3 & -\alpha_2 & \alpha_1 & \beta_0\end{pmatrix}$ and the evolved vector $\vec{\lambda}(t)=(\lambda_0,\lambda_1,\lambda_2,\lambda_3)^{\mathrm{T}}$ describing the evolved state at any time. In general, the state vector $\vec{\lambda}(t)$ is given by $\vec{\lambda}(t)=e^{\hat{\zeta}t}\cdot\vec{\lambda}(0)$ and $\vec{\lambda}(0)$ represents the vector of an initial state. With respect to the diagonal form of $\hat{\zeta}=\sum_{i=1}^{4}h_i|v_i\rangle\langle v_i|$, the exponential function is given by $e^{\hat{\zeta}t}=\hat{V}\hat{R}\hat{V}^{-1}$ and the i-th column of the transformation matrix $\hat{V}$ is the corresponding eigenvector $|v_i\rangle$. The diagonal matrix $\hat{R}=\mathrm{diag}(e^{h_1t},e^{h_2t},e^{h_3t},e^{h_4t})$ is determined by the eigenvalues of the mapping matrix.

To study quantum dynamics of a non-Hermitian system, we take into account a single-ion system where the PT-symmetric Hamiltonian with balanced gain and loss has been realized in the experiment[19]. The experimental setup involves a trapped 40Ca$^+$ ion in a magnetic field. Initially prepared in the ground state $|g\rangle=|^2S_{-1/2}\rangle$, the system is excited to an upper state $|e\rangle=|^2D_{5/2}\rangle$ using a laser with a wavelength of 729 nm. Another laser at 854 nm induces a controllable loss in $|e\rangle$ which is coupled to a short-lived level $|^2P_{3/2}\rangle$ and quickly decays to the state $|d\rangle=|^2S_{1/2}\rangle$. By means of laser driving, the coherent energy transition between $|e\rangle$ and $|g\rangle$ can be performed and the excited state undergoes the tunable loss, as shown in Figure 1. From the viewpoint of open quantum systems, the states of $|e\rangle$ and $|g\rangle$ experience the interaction with the dissipation environment $|d\rangle$ at a loss rate.

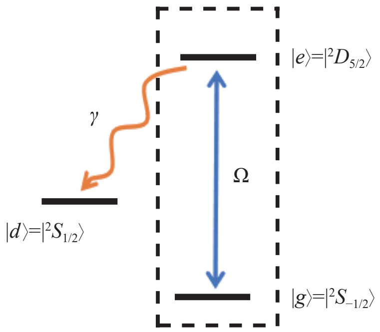


Fig. 1 The two levels of the ion system with balanced gain and loss

In this case, the effective two-level system is described by the non-Hermitian Hamiltonian,

$$\hat{H}_{\mathrm{eff}}=\frac{\Omega}{2}\sigma_1-i\frac{\gamma}{2}\sigma_3-i\frac{\gamma}{2}I\quad, \tag{5}$$

where γ is the gain-loss rate and Ω is the coherent coupling. Correspondingly, $\vec{\alpha}=\left(0,\frac{\Omega}{2},0,0\right)^{\mathrm{T}}$, $\vec{\beta}=\left(-\frac{\gamma}{2},0,0,-\frac{\gamma}{2}\right)^{\mathrm{T}}$ can characterize Hermitian part and anti-Hermitian one of the non-Hermitian Hamiltonian in the trapped ion system. Additionally, some experimental setups[20] of non-Hermitian system are realized in the field of optics and photonics.

4 Quantum parameter estimation as a signature of the exceptional points of a non-Hermitian system

Quantum phase transition in non-Hermitian systems is traditionally characterized by the energy spectrum of the Hamiltonian. Like the non-Hermitian Hamiltonian of Eq. (6), the PT symmetric part $\hat{H}_{\mathrm{PT}}=\frac{\Omega}{2}\sigma_1-i\frac{\gamma}{2}\sigma_3$ of the Hamiltonian has two eigenvalues of $E_{1,2}=\pm\frac{1}{2}\sqrt{\Omega^2-\gamma^2}$ where the region of $\frac{\gamma}{\Omega}<1$ corresponds to the unbroken phase of PT symmetry and $\frac{\gamma}{\Omega}>1$ represents the PT symmetry broken phase. The criticality occurs at an EP of $\gamma=\Omega$ due to the interplay between the gain-loss rate and coherent coupling.

Recently, some feasible approaches in quantum information theory can be used to describe the criticality at the exceptional points of non-Hermitian systems, such as quantum entanglement, quantum correlation and others. It is interesting to study the physical relation between quantum criticality and quantum parameter estimation in non-Hermitian systems. In the experimental condition, we can obtain the dynamics of the non-Hermitian single ion system by using Eq. (5). And the eigenvalues of $\hat{\zeta}$ are $h_1=-\gamma$, $h_2=-\gamma$, $h_{3,4}=-\gamma\pm K$ and $K=\sqrt{\gamma^2-\Omega^2}$. The eigenvectors are given as $|v_1\rangle=\begin{pmatrix}0\\1\\0\\0\end{pmatrix}$, $|v_2\rangle=\begin{pmatrix}1\\0\\\lambda/\Omega\\0\end{pmatrix}$, $|v_{3,4}\rangle=\begin{pmatrix}1\\0\\\Omega/\gamma\\\mp K/\gamma\end{pmatrix}$ respectively. The initial state as a function of estimated parameters can be expressed as,

$$\rho(0)=\begin{pmatrix}\cos^2\left(\frac{\theta}{2}\right) & \sin\frac{\theta}{2}\cos\frac{\theta}{2}e^{-i\phi}\\ \sin\frac{\theta}{2}\cos\frac{\theta}{2}e^{i\phi} & \sin^2\left(\frac{\theta}{2}\right)\end{pmatrix}, \quad (6)$$

where $\rho(0)=|\phi(0)\rangle\langle\phi(0)|, |\phi(0)\rangle=\cos\frac{\theta}{2}|e\rangle+\sin\frac{\theta}{2}$ $e^{i\varphi}|g\rangle$. The estimated parameters of θ,φ are related to the directly measured density matrix elements. The equivalent form of the initial state vector is written as $\vec{\lambda}(0)=(1,\sin\theta\cos\varphi,\sin\theta\sin\varphi,\cos\theta)^{\mathrm{T}}$. At any time, the density matrix $\rho(t)$ can be described by the state vector $\vec{\lambda}(t)$ in the form of

$$\begin{aligned}
\lambda_0(t)&=e^{-\gamma t}\left[\frac{\gamma^2}{K^2}\cosh(Kt)-\frac{\Omega^2}{K^2}+\left(\frac{\gamma\Omega}{K^2}\cosh(Kt)+\frac{\gamma\Omega}{K^2}\right)\sin\theta\sin\phi-\frac{\gamma}{K}\sinh(Kt)\cos\theta\right],\\
\lambda_1(t)&=e^{-\gamma t}\sin\theta\cos\phi,\\
\lambda_2(t)&=e^{-\gamma t}\left[\frac{\gamma\Omega}{K^2}\cosh(Kt)-\frac{\gamma\Omega}{K^2}+\left(\frac{\Omega^2}{K^2}\cosh(Kt)+\frac{\gamma^2}{K^2}\right)\sin\theta\sin\phi-\frac{\Omega}{K}\sinh(Kt)\cos\theta\right],\\
\lambda_3(t)&=e^{-\gamma t}\left[-\frac{\gamma}{K}\sinh(Kt)-\frac{\Omega}{K}\sinh(Kt)\sin\theta\sin\phi+\cosh(Kt)\cos\theta\right].
\end{aligned} \quad (7)$$

Without loss of generality, we consider the estimation precision of the phase parameter $\phi=\arctan\left(\frac{\lambda_2}{\lambda_1}\right)$ which is bounded by the QFI,

$$F_\phi=\left|\partial_\phi\vec{B}\right|^2+(\vec{B}\cdot\partial_\phi\vec{B})^2/(1-|\vec{B}|^2), \quad (8)$$

here the vector is given by $\vec{B}=\frac{1}{\lambda_0}(\lambda_1,\lambda_2,\lambda_3)^{\mathrm{T}}$ which is dependent on the normalized density matrix $\tilde{\rho}_\phi=\rho_\phi/Tr(\rho_\phi)$ with the estimated phase parameter.

To demonstrate the connection between QFI and quantum phase transition at EP, we explore the dynamics of QFI as a function of the ratio γ/Ω, which is shown in Fig. 2(a) (color online). It is seen that the values of QFI approach to the maximum near the EP of $\gamma=\Omega$ at the early stage of the evolution. This result proves that the non-classical effects are amplified in the vicinity of the EP. In the unbroken region of PT symmetry, the oscillatory behavior of QFI can occur in Fig. 2(b). In fact, a non-Hermitian system can be referred to as one open system. The discontinuous revival behavior can arise from the non-Markovian dynamics which is characterized as information retrieval from the environment to the system. However, the QFI undergoes the monotonically decreasing evolution in the broken phase as shown in Fig. 2(c). The unidirectional information flow from the system to the environment leads to the monotonic decaying of QFI. Therefore, the EP masks the oscillation and monotonic declination of quantum Fisher information. The two different ways of the evolution of QFI can be regarded as a signature of quantum phase transition. Moreover, the more accurate estimation can be obtained in the unbroken phase of PT symmetry.

To furthermore study the effects of the balanced gain and loss on estimation precisions, we also illustrate the short-time behavior of QFI with respect to all possible values of the phase parameter. From Fig. 3 (color online), the contour plot of QFI as a function of $\frac{\gamma}{\Omega}$ and φ is shown in the condition of $\Omega=2\pi\times32$ kHz, $\theta=\pi/2$, $t=19$ μs. We can observe the values of QFI arrive at some peaks in the vicinity of the EP of $\frac{\gamma}{\Omega}=1$. It is demonstrated that for some certain values of $\varphi=n\pi(n=0,1,2\cdots)$, the maximal values of the QFI are obtained at the exceptional point. This result suggests that near the EP, there is an enhanced availability of quantum estimation precision. The QFI is sensitive to quantum criticality at the EP.

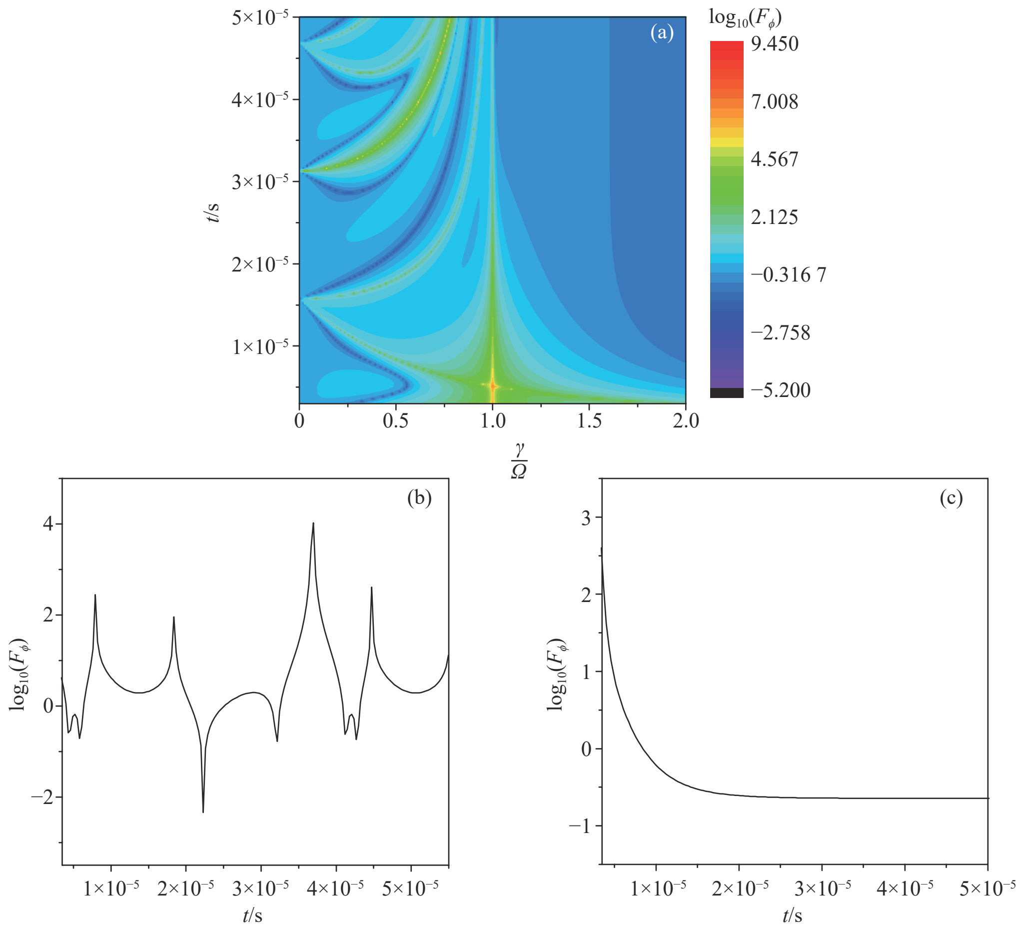


Fig. 2 The dynamics of QFI is plotted as a function of the ratio γ/Ω in the condition of $\Omega = 2\pi\times 32$ kHz, $\theta = \pi/2, \phi = 0$. (a) The contour plot; (b) the oscillation illustrated in the PT symmetry unbroken region of $\frac{\gamma}{\Omega} = 0.43$; (c) the decaying behavior in the broken phase of $\frac{\gamma}{\Omega} = 1.5$

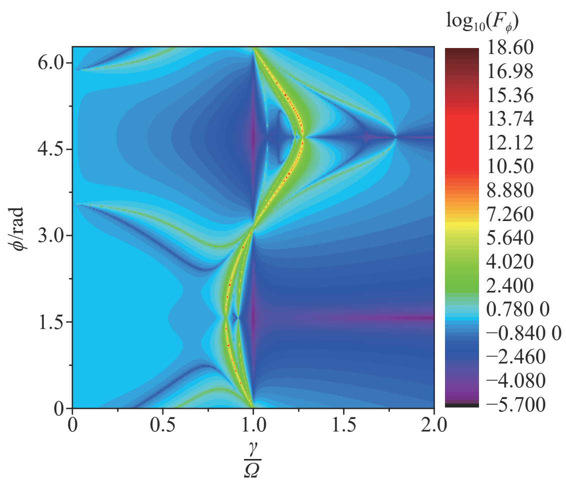


Fig. 3 The contour plot of QFI is illustrated at a short time interval.

It is also demonstrated that the non-Hermitian QFI about parameter estimation can be regarded as one feasible witness to determine the critical points of the system.

5 Quantum information witness of PT phase transition

In the study of the dynamics of quantum systems, people often focus on the evolution of some facets of quantum information including quantum nonlocality. We expect to find more evidences to characterize quantum phase transition in a non-Hermitian system, from the viewpoint of quantum information. In the following, the dynamical behavior of quantum entropy and quantum coherence are examined to demonstrate the effects of quantum information facets on quantum criticality at the EP.

It is known that quantum entropy can reveal the flow and distribution of quantum information within the system. The characteristics of the von Neumann entropy can be obtained from the dynamics of the non-Hermitian system. For the non-Hermitian system, quantum entropy is defined by the normalized density matrix[21,22]:

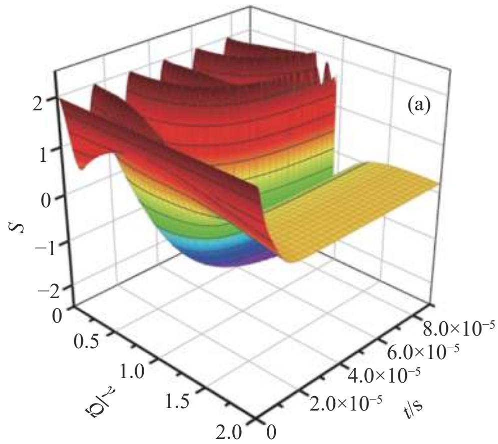
$$S(\tilde{\rho}) = -Tr[\tilde{\rho}\ln\tilde{\rho}] \quad . \tag{9}$$

When the single ion system is considered, the normalized density matrix is written as

$$\tilde{\rho}(t) = \begin{pmatrix} \frac{1}{2} + \frac{\lambda_3}{2\lambda_0} & \frac{\lambda_1 - i\lambda_2}{2\lambda_0} \\ \frac{\lambda_1 + i\lambda_2}{2\lambda_0} & \frac{1}{2} - \frac{\lambda_3}{2\lambda_0} \end{pmatrix} \quad . \tag{10}$$

Here the initial state $\rho(0)$ is chosen as the form of Eq. (6). The dynamical behavior of quantum entropy can be plotted in Fig. 4(a) (color online) when the parameters are $\Omega = 2\pi \times 32$ kHz, $\theta = \pi/2, \varphi = 0$. It is shown that in the region of $\frac{\gamma}{\Omega} < 1$, quantum entropy exhibits periodic oscillations with time. In contrast, for the PT symmetry broken region of $\frac{\gamma}{\Omega} > 1$, the entropy gradually decreases and approaches to a stable value. From Fig. 4(b), we can observe that the time derivative dS/dt suddenly changes near the EP. The time is set as $t = 66.8$ μs.

The Fig. 4(b) shows the first derivative of entropy with respect to the ratio of $\frac{\gamma}{\Omega}$. From the figure, we can clearly distinguish three different regions. In the PTS region, the entropy has a maximum value implies the presence of substantial quantum resources. If the value of $\frac{\gamma}{\Omega}$ approaches to the EP, the time derivative of entropy suddenly changes to zero from negative values. The result also demonstrates that the maximum of quantum entropy can be accessed in the vicinity of the EP. It is proven that the dynamics of quantum entropy can be used to describe the critical phenomena of the non-Hermitian system.



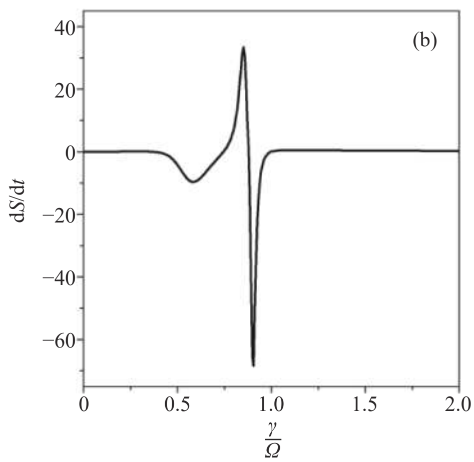


Fig. 4　(a) The evolution of quantum entropy. (b) The change of dS/dt

Meanwhile, quantum coherence emerges when quantum states exist in the form of the superposition. It serves as one kind of fundamental resources in quantum information technology.

According to the definition of l_1-norm[23-24], quantum coherence can be calculated as $C(\tilde{\rho}) = \sum_{i\neq j}|\tilde{\rho}_{ij}|$ and determined by the off-diagonal elements of the normalized density matrix. The expression of quantum coherence for the non-Hermitian ion system can be given by

$$C_{l1} = \frac{\sqrt{\lambda_1^2+\lambda_2^2}}{|\lambda_0|} \quad . \tag{11}$$

The dynamical behavior of quantum coherence as a function of $\frac{\gamma}{\Omega}$ can be shown in Fig. 5 (color online) when the parameters are $\Omega = 2\pi \times 32$ kHz, $\theta = \pi/2, \phi = 0$.

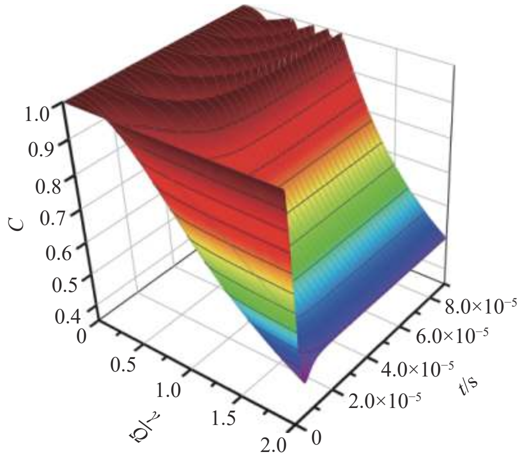


Fig. 5 The evolution of quantum coherence

It is seen that the high values of coherence occur in an oscillatory manner in the PT symmetry unbroken region. With the increase of the loss γ, quantum coherence decreases monotonically in the PT symmetry broken region. The EP marks the two different kinds of behavior from the viewpoint of quantum coherence.

6 Conclusion

Utilizing quantum metrology techniques, we provide a witness of quantum Fisher information to characterize the phase transition of a non-Hermitian system. The state vector of the normalized density matrix is used to express the dynamics of the non-Hermitian QFI. The single trapped ion system can be applied to our scheme. The two different types of dynamical behaviors are observed near the exceptional point. In the PT symmetry unbroken region, the oscillatory values of the QFI keep large, which represent the high precision of quantum parameter estimation. On the other hand, the monotonical declination of the QFI is demonstrated in the PT symmetry broken region. With the increase of the loss, the QFI gradually decreases towards a steady value which is dependent on the equilibrium state. The sudden change in the evolution of QFI at EP can be regarded as a signature of quantum criticality of the non-Hermitian system. In addition, we have considered the dynamical behavior of quantum entropy and coherence. From the viewpoint of quantum information, the discontinuous behavior can also be demonstrated at the EP. Much more quantum resources like quantum entropy and coherence can be obtained near the EP, which is beneficial for quantum information processing. Our work not only may provide a new way to enhancing parameter estimation precision but also discover the physical relation between quantum information and quantum criticality in a non-Hermitian system.

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